A Comparative Study on Probabilistic Optimization Methods for Electromagnetic Design

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A reliability-based roust design optimization (RBRDO) is developed to ensure the product quality as well as the confidence in product reliability of electromagnetic devices. In the method, the first two statistical moments, mean and variance, of a quality loss function is estimated by the univariate dimension reduction method (DRM), while desired probabilistic constraint conditions is assessed by the first-order reliability analysis method. For better understanding between probabilistic optimization methodologies, three different formulations of reliability-based design optimization (RBDO), robust design optimization (RDO) and RBRDO are presented and compared with each other. A simple mathematical design problem is tested to demonstrate the features of the three methods and to examine their numerical efficiency.

***Index Terms*—Electromagnetics, optimization, reliability theory, robustness.**

I. Introduction

In out community, engineers have paid increasing attention to probabilistic optimization, which enables designers to quantitatively take into account uncertainties observed in electromagnetic (EM) properties. In accordance with the design purpose to be achieved, the probabilistic optimization can be classified into three methodologies: RBDO, RDO and RBRDO [1]-[2]. In recent years, the first two methods have been successfully applied to EM design problems. RBDO involving probabilistic constraint conditions is a method to enhance the confidence in product reliability at a given probabilistic level [3]. On the other hand, the RDO method is to optimize the mean of the product quality loss function and to minimize its variance simultaneously [4]-[5]. Since both optimization methods make use of probabilistic information on design variables or parameters, now, it is natural that the two different methodologies are integrated to develop a RBRDO method for EM design.

II. Three Formulations for Probabilistic Optimization

Unlike deterministic design optimization (DDO) without considering uncertainties, a probabilistic optimization problem is generally expressed in terms of the quality loss function or probabilistic constraint functions. Consequently, the most important task of a probabilistic design is uncertainty analysis associated with the loss or constraint functions. In this section, three different formulations for probabilistic optimization are presented and compared with each other.

A. Reliability-Based Design Optimization

A typical RBDO formulation is given by

$$\begin{aligned} &\text{minimize} \quad h(\mathbf{d}) \\ &\text{subject to} \quad P_F(g_i(\mathbf{X})>0) \le P_{t,i}, \quad i=1,2,\ldots nc \end{aligned} \tag{1}$$

where h is the cost/objective function, and $\mathbf{d}$ is the vector of design variables defined by $\mathbf{d}=\boldsymbol{\mu}(\mathbf{X})$, where $\boldsymbol{\mu}$ denotes the mean of design random variables $\mathbf{X}$. The symbol g_i is the ith function of nc constraints, $P_F(\cdot)$ is the probability of failure for the infeasible condition ($g_i > 0$), and $P_{t,i}$ is the ith target value for ensuring the confidence/reliability level ($1-P_{t,i}$) of g_i. After all, the RBDO design focuses on making probabilistic constraint conditions (i.e. product reliability) satisfied at a desired confidence level while minimizing the given cost function.

B. Roust Design Optimization

A conventional RDO problem is mathematically written by

$$\begin{aligned} &\text{minimize} \quad f(\mu_h, \sigma_h^2), \quad h(\mathbf{X};\mathbf{d}) \\ &\text{subject to} \quad g_i(\mathbf{d}) \le 0, \quad i=1,2,\ldots nc \end{aligned} \tag{2}$$

where f is the quality loss function consisting of the two statistical moments, mean μ_h and variance σ_h^2, of the cost function h. Since the loss function is usually expressed in terms of the sum of μ_h and σ_h^2 multiplied by certain weight factors, it belongs to a bi-objective optimization problem. The goal of RDO is to find the most insensitive design (i.e. product quality/robustness) to the variation of uncertainties within a feasible design space ($g_i \le 0$).

C. Reliability-Based Roust Design Optimization

In a RBRDO formulation, the product quality loss function is minimized subject to probabilistic constraints as follows.

$$\begin{aligned} &\text{minimize} \quad f(\mu_h, \sigma_h^2), \quad h(\mathbf{X};\mathbf{d}) \\ &\text{subject to} \quad P_F(g_i(\mathbf{X})>0) \le P_{t,i}, \quad i=1,2,\ldots nc. \end{aligned} \tag{3}$$

To solve (3), both RBDO and RDO approaches have to be combined into a single numerical model. Such nested optimization problems require a relatively expensive computational cost. Nevertheless, RBRDO seems one of very promising and challenging problems because it can lead to a very robust and reliable product design against the variation of input variables and parameters.

In this paper, an attempt to tackle the complicated problem of (3) was made for EM design. Fig. 1 shows the program architecture of a newly developed RBRDO method. The univariate DRM in [5] was employed to estimate the statistical moments of the quality loss function and their sensitivities, whereas the performance measure approach (PMA) in [3] was adopted to assess probability constraint functions and their sensitivities. The sequential quadratic programming algorithm was utilized for handling a constrained optimization problem.

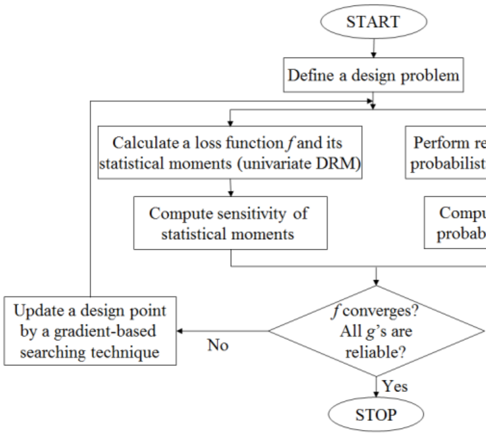

Fig. 1. Flowchart of the proposed RBRDO method.

III. Results

To highlight individual features of probabilistic optimization methods, a two-dimensional (2D) mathematical design problem with an analytical solution is considered. A RBRDO formulation of the test problem is given by

$$\begin{aligned} &\text{minimize} \quad f(\mu_h, \sigma_h^2) = w_1(\mu_h/\mu_{h0})^2 + w_2(\sigma_h^2/\sigma_{h0}^2)^2 \\ &\qquad\qquad h(\mathbf{X}) = (x_1-4)^3 + (x_1-3)^2 + (x_2-5)^2 + 10 \\ &\text{subject to} \quad P_F(g_1(\mathbf{X}) = -x_1 - x_2 + 7.45 > 0) \le P_t \\ &\qquad\qquad P_F(g_2(\mathbf{X}) : x_i < 1 \text{ or } x_i > 10) \le P_t \quad i = 1,2 \end{aligned} \tag{4}$$

where w_1 and w_2 are weight factors of 0.5 and 1.0 respectively. The symbols, μ_{h0} and σ_{h0}^2, are the nominal values (mean and variance) of the cost function h calculated at an initial point (8, 8). The random variable vector **X** is assumed to have a normal probability distribution with a standard deviation of 0.4. The target probability of failure P_t is set to be 0.13% for two constraints (i.e. reliability of 99.87%)

For comparison, a conventional DDO was first applied to a modified formulation of (4), where h is minimized subject to $g_1 \le 0$ and $g_2 : 1 \le x_i \le 0$ without considering the randomness of design variables. Starting with the same initial point, the problem was then solved in accordance with the three different formulations, (1), (2) and (4), for probabilistic optimization: PMA-based RBDO, univariate DRM-based RDO and proposed RBRDO methods.

Four different optima are compared with each other in Fig. 2, where contour lines of h and constraint boundaries (g_i=0) are depicted as well. As expected, the DDO point is located at the corner of the first constraint boundary (g_1=0) and the lower bound (x_1=1) of random variables marked in dotted line. The RDO point is observed on the first constraint boundary. Meanwhile, RBDO and RBRDO points are spatially apart from the two constraint boundaries in order to satisfy the prescribed probability constraint conditions. Table I presents performance indicators between four different designs. To examine the accuracy of the univariate DRM using five quadrature points, the variance values of h estimated at RDO and RBRDO points are compared with exact ones, which are calculated analytically. As seen in the Table, the estimated values show a good agreement to the exact ones. Even though RDO has a larger variance value than RBRDO, it meets the design goal of the smallest value of the quality loss function *f*. In terms of the probabilistic conditions, DDO point and RDO points have a nearly 50% failure probability for g_1 or g_2 when engaged in the randomness of design variables. Whereas, both RBDO and RBRDO optima fulfill the failure probability of less than 0.13% given in (4). From the results, it is obvious that the proposed RBRDO method yields a robust and reliable solution against the variation of design variables even though it requires the largest number of function calls compared to other methods.

The extended version of the paper will present a practical EM design problem, where a robust and reliable design of a BLDC motor is attempted.

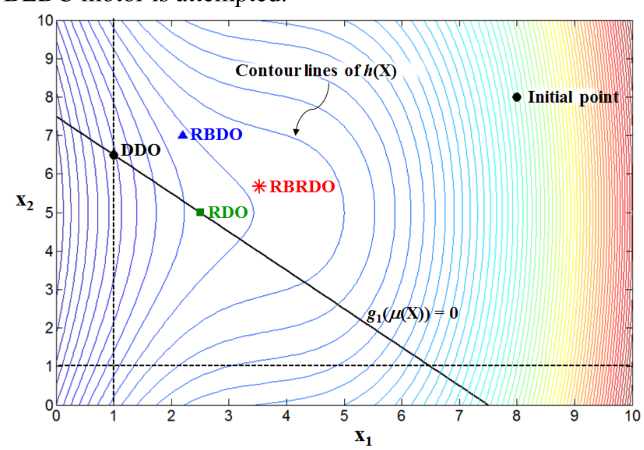

Fig. 2. Contour of the cost function $h(\mathbf{X})$ and different design points.

TABLE I
Performance Indicators at Four Different Designs

		DDO	RBDO	RDO	RBRDO
d_1		1.00	2.20	2.50	3.53
d_2		6.50	7.00	5.00	5.67
Mean (h)		-11.87	8.25	6.46	10.71
Variance (h)	Estimated	-	-	6.92	1.13
	Exact	-	-	6.95	1.15
Quality loss function f		-	-	0.002	0.005
$P_F(g_1)$		50.16%	0.12%	50.06%	0.12%
$P_F(g_2)$		50.04%	0.13%	0.00%	0.00%
Iterations/Function calls		6/22	3/81	9/231	9/448

* P_F was recalculated by Monte Carlo simulation with 500,000 samples.

IV. References

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